Interest Arbitrage Using Near Term Futures on 5-year Treasury Bond

Exploiting an Inverted / Flat yield Curve

Daily Treasury yield Curve Rates
1/06/2006 – from us. treasury website

Date	1 month	3 months	6 months	5 year
1/06/2006	4.06	4.22	4.39	4.32

Short term yield 6 months > yield on 5-year Treasuries

Potential Action: Short the Spot market on a $100,000 5-year T Bond
go Long the near term 5-year Futures
Unwind the position 3 months later to earn a profit on the relative pricing difference

Note: This Cash & Carry arbitrage depends on the relative difference in pricing caused by an unusual inverted yield curve situation.

It is important that the security in the cash (spot) market match up with the underlying security for the futures contract.

5-year U.S. Treasury Note Futures on 1/6/2006

Expiration	Open	Close	
March 15, 2006	106'165	106'170	} from CBOT

U.S. Treasury Bond Note Price/yield
from Yahoo Bond Screener

U.S Treas. Note 3.5% due Jan 15, 2011

Price	Coupon	Yield
107.39	3.5	1.945

Chicago Board of Trade

5 Year U.S. Treasury Notes Futures (ZF)

Delayed 10 minute data as of January 09, 2006 20:41 CST

Quotes Settlement Daily Vol Time & Sales Volatility Historical Data Spreads

Open Auction Electronic Combinations Real-Time Quotes

Exp	Last 1 Last 2	Net Chg	Open	High	Low	Close	Settle	Prev Settle	Hi/Lo Limit	ETS Vol
06Mar	106'170 20:39	+0'015	106'165 18:00	106'180 19:29	106'160 18:02	106'170 20:39		106'155		3213

Table generated January 09, 2006 20:41 CST = Chart = Option

Product News

- Termination of Trading in CBOT 10-Year Municipal Note Index Futures - 12.20.2005
- Pilot Program - Algorithm in 2 Year Treasury futures - 12.19.2005
- October 2005 CBOT Monthly Interest Rate Update: A Global Trading Summary - 11.28.2005
- Market Maker Programs for Options on Two-Year U.S. Treasury Note Futures - 11.17.2005

Product Strategies

- The CBOT TUT Spread - 09.21.2005
- The CME/CBOT Common Clearing Link - 09.04.2003
- Trading the TUT Spread: Capitalizing on Changes in Yield Curve - 08.29.2003
- An Outline of Fundamental Economic Reports - 06.16.2003
- Simple Treasury Duration Adjustment - 05.28.2003
- Yield Curve Shift Makes Trading Opportunities - 05.28.2003
- Trading 10-Year Term TED Spreads - 05.27.2003
- A Futures Overlay to Reduce Cash Drag - 05.27.2003
- The 5-Year Term TED Spread - 05.27.2003
- Enhancing a Bond Yield with Calls on CBOT Swap Futures - 05.27.2003

Bond Center

Bond Center > Bond Screener > Bond Screener Results

From Yahoo Bond Screener 1/9/2006

BOND SCREENER RESULTS

	Type	Issue	Price	Coupon(%)	Maturity	YTM(%)	Current Yield(%)	Rating	Callable
ok	Treas	T-NOTE 3.500 15-Jan-2011	107.39	3.500	15-Jan-2011	1.945	3.259	AAA	No
	Treas	T-NOTE 5.000 15-Feb-2011	103.27	5.000	15-Feb-2011	4.278	4.841	AAA	No
	Treas	T-BOND 13.875 15-May-2011	103.38	13.875	15-May-2011	12.966	13.421	AAA	Yes
	Treas	T-NOTE 5.000 15-Aug-2011	103.50	5.000	15-Aug-2011	4.288	4.830	AAA	No
	Treas	T-BOND 14.000 15-Nov-2011	108.12	14.000	15-Nov-2011	12.014	12.948	AAA	Yes
	Treas	T-NOTE 3.375 15-Jan-2012	107.92	3.375	15-Jan-2012	1.970	3.127	AAA	No
	Treas	T-NOTE 4.875 15-Feb-2012	102.92	4.875	15-Feb-2012	4.323	4.736	AAA	No
	Treas	T-NOTE 3.000 15-	106.14	3.000	15-Jul-2012	1.990	2.826	AAA	No

Cash & Carry Arbitrage Transaction

Assume the coupon interest issue is not relevant because Shorting the Spot bond at the beginning of a coupon interest period $\Rightarrow$ little accrued interest.

Buying the Futures Contract at the beginning of a coupon interest period means the price going into the future will reflect coupon interest.

1/6/2006 — 3/15/2006

Short (Sell) \$100,000 U.S. Treas 3.5% Coupon Bond due 2011 take proceeds → Buy (long) \$100,000 U.S. Treasury Bond

Buy 1 (go long) 1 Futures Contract 5-year T Note expiring 3/15/2006 → Sell (Short) the Futures Contract take proceeds and

• On 1/6/2006

Short the U.S. Treas Bond @ $107.39 / $100 face Amt

⇒ $107.39 x 1000 = $107,390 proceeds

take Cash to Buy 1 Future Contract @ 106.17

⇒ $106 + 17/32 = 106.53125$

Purchase price · 106.53125 x 1000 = $106,531.25

• On 3/15/2006 you sell the Future Contract take the proceeds to cover the Short Bond position.

The difference between the initial spot sale $107,390 less the cost of the Future Contract

$106,531.25

858.75 represents the advantage in the position

•

Setting limits on option premiums

Arbitrage occurs whenever two similiar assets sell for widely different prices in two markets.

Because financial assets tend to be uniform & homogeneous in structure – arbitrage opportunities become readily apparent – which is why they don't last long.

Financial markets allow one to combine different securities together to produce identical payment patterns as another security – The combined securities are called **Synthetic Securities.**

These other alternative securities force strict pricing limits among securities to prevent long-term arbitrage.

Forward rates (or yields) are implied by current cash yields based on term structure of interest rates.

$$(1+r_{0,L})^{L} = (1+r_{0,S})^{S}\,(1+r_{S,L})^{L-S}$$

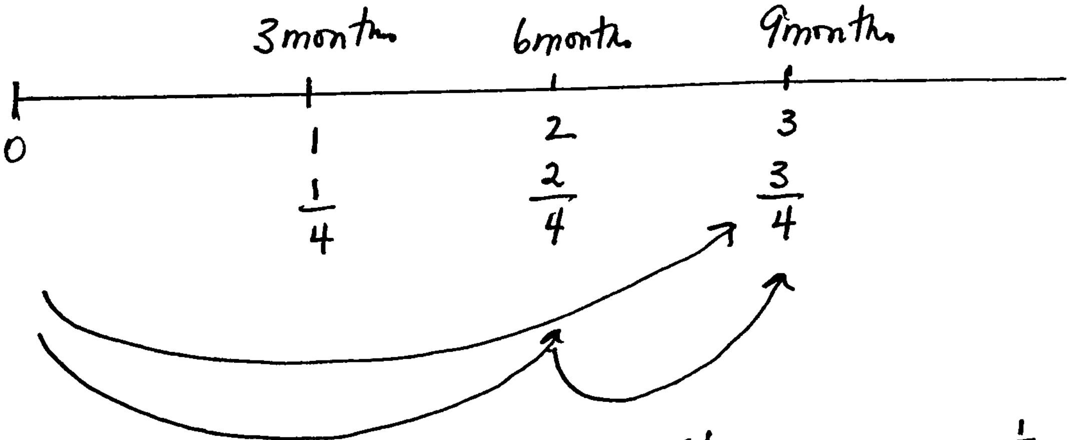
$r_{0,L}$ = Cash yield on long-term security - with maturity L
$r_{0,S}$ = Cash yield on short-term security - with maturity S
$r_{S,L}$ = implied forward rate between S and L

E.g. Assume a cash yield on a 1-year T-Bill is 4.5%. A two year Treasury Note yields an average annual return of 4.7% (positive yield curve). What must be earned in the T-Bill market in year 2 to be an equivalent investment to the Treasury Note?

$$(1+.047)^{2} = (1+.045)^{1}\,(1+r_{S,1})^{1}$$

$$4.9\% = \frac{(1.047)^{2}}{1.045} - 1 = r_{S,1}$$

E.G. What is the implied forward rate from 6 to 9 months, if a 9 month T-Bill yields 6.5% and a six month T-Bill yields 6.25%?



$$\left(1+\frac{.065}{4}\right)^{3/4} = \left(1+\frac{.0625}{4}\right)^{2/4}\left(1+\frac{r}{4}\right)^{\frac{1}{4}}$$

$$(1+.01625)^{.75} = (1+.015625)^{.5}\left(1+\frac{r}{4}\right)^{\frac{1}{4}}$$

$$1.012162910 = (1.007782219)\left(1+\frac{r}{4}\right)^{\frac{1}{4}}$$

$$(1.004346863) = \left(1+\frac{r}{4}\right)^{\frac{1}{4}}$$

$$(1.004346863)^4 = 1+\frac{r}{4}$$

$$1.017501153 = 1+\frac{r}{4}$$

$$4(.017501153) = r$$

$$r = .070004612 \approx \underline{\underline{7\%}}$$

Theorem 1 [lower bound, min value for Call premium]

At expiration, the Call premium can be no smaller than either 0 or the difference between the underlying stock price and the fixed exercise price.

$$C_t = \text{Max}\{0, S_t - X\}$$

C_t = Call premium that exists on the expiration day t

S_t = Stock price on the expiration date t

X = fixed exercise price that remains constant over the life of the contract all the way to the expiration date t.

A zero call premium ⇒ out of the money call
the stock price at expiration is below the exercise price. If you exercised the option, paid the exercise price and bought the stock ⇒ you would be paying more than the stock was currently worth.

Abbott Laboratories (ABT)

At 4:00PM ET: **42.00**

Options

January 10, 2006

Options Expire Jan. 20, 2006

Get **Options** for:

View By Expiration: **Jan 06** | Feb 06 | May 06 | Aug 06 | Jan 07 | Jan 08

CALL OPTIONS

Expire at close Fri, Jan 20, 2006

in the money

Strike	Symbol	Last	Chg	Bid	Ask	Vol	Open Int
20.00	WKPAD.X	**26.40**	↑3.60	26.30	26.70	302	88
30.00	ABTAF.X	**11.90**	↑2.50	11.90	12.10	942	169
30.00	WKPAF.X	**16.40**	↑3.50	16.30	16.70	4,542	368
35.00	ABTAG.X	**6.90**		6.90	7.10	7,682	634
37.50	ABTAU.X	**4.40**		4.40	4.60	11,907	943
40.00	ABTAH.X	**2.00**		1.95	2.05	23,334	9,105

View By Expiration: **Jan 06** | Feb 06 | May 06 | Aug 06 | Jan 07 | Jan 08

out of the money

47.50	ABTAW.X	**0.05**	**0.00**	N/A	0.05	3	5,062
50.00	ABTAJ.X	**0.05**	**0.00**	N/A	0.05	2	16,612
50.00	WKPAJ.X	**0.10**	**0.00**	N/A	0.15	21	925
55.00	ABTAK.X	**0.05**	**0.00**	N/A	0.05	4	3,530

ET: 42.00

Options for:

20, 2006

Open Int

88

169

368

634

943

9,105

E.S. Consider the following call option information on Home Depot (HD):

HD Stock price → \$43

Exercise (Strike) price on Call option → \$40

Call premium → \$4.50

What type of trading strategy could be used to produce an arbitrage profit?

Note:

Stock price − Exercise price < Call premium

43 − 40 < 4.50

Out of the money Call

⇒ Buy the stock at \$43

Write a Call

Sell (pick up) Call premium — \$4.50

If the stock rises above \$ 43 + 4.50 = \$47.50 it will get called away

Theorem 2 [upper bound for the Call premium]

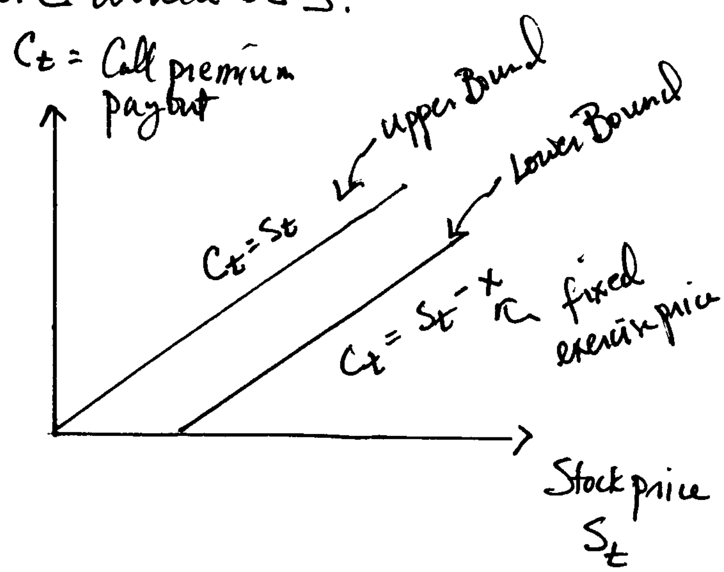
The Call premium Can never be more than the value of the underlying Stock.

Polar Case: lowest possible strike price is 0.

If $X = 0$, $C = S - X \Rightarrow C = S$.

Therefore, the highest value for C would be S.

$C_t = S_t - X$



Theorem 3 [You pay a premium for more time]

Given two options that are identical in terms of having the Same exercise price and underlying asset, the option with longer time to expiration will have a higher premium.

If $T_1 > T_2$ then $C_1 > C_2$

Theorem 4 [There is an inverse rel^n ship between the exercise price and the Call premium]

Given two options that are the Same in all respects, except one has a higher exercise price, then it must have a lower call premium.

When $X_1 > X_2$ then $C_1 < C_2$.

If $C_t = S_t - X$, then let $S_t = S$ the stock price whenever $X\uparrow$, $C_t = S - X\uparrow$ will decline.

Theorem 5 [lower bound on the call premium on a present value basis]

Prior to expiration, the call premium (C_t) will not be less than either 0. Or the current stock price minus the present value of the exercise price X.

$$C_t \geq \text{Max}\{0, S_t - X e^{-rt}\}$$

If the call is out of the money $C_t = 0$, the call is worthless.

S_t = present value of the cash flows from the call option representing the stock

↑ Current price of the stock

Expiration

S_t $\qquad$ e^{-rt} $\qquad$ X payment to acquire the stock

↑ Stock price today

= C_t Call premium
+ present value of the exercise price

Theorem 6 [The Call option provide unlimited upside return while limiting downside loss]

The higher (lower) the risk of the underlying asset the higher (lower) the Call premium.

$$\sigma_1 > \sigma_2 \Rightarrow C_1 > C_2$$

Higher st.dev. of the underlying stock — greater the Call premium reflecting higher return potential

Theorem 7 [In the absence of a dividend time matters]

When a stock does not pay a dividend during the life of the option, then early exercise is never optimal

quire

Theorem 8 [Dividend paying Stocks may present an opportunity to exercise early the call option]

When a stock pays a dividend during the life of the option, then early exercise may be optimal if the dividend plus interest exceeds the interest that could be earned on the exercise price.

$$D e^{r't'} > X \{ e^{r''t''} - 1 \}$$

where r' = forward rate from the time the dividend is paid until expiration

t' = the number of days between the expiration date and the time the dividend is paid

r'' = forward rate during the period one day before the ex dividend date and the expiration date

t'' = the number of days one day before the ex dividend date and the expiration date.

Explanation

The call option conveys the right to purchase the underlying stock at a fixed price — namely, the exercise price X.
If the stock pays a dividend over the life of the call you have 2 alternatives:

1. you can exercise the call one day before the exdividend date, paying X and receiving the dividend
2. you can do nothing

Note: In either case you will control the stock over the period of the option (either as acquired stock or through the call).

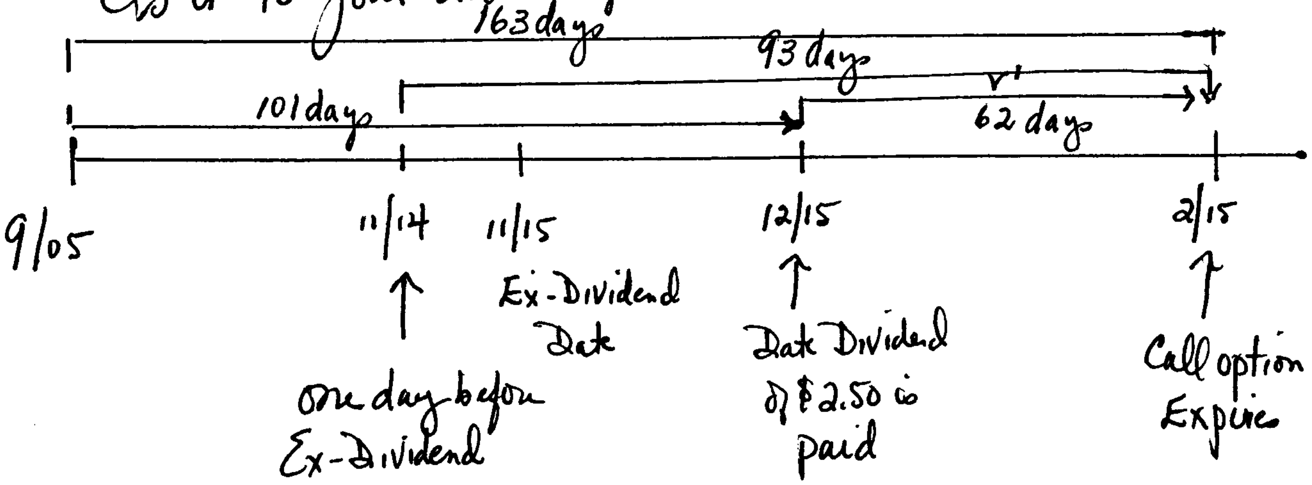
To determine whether it is better to take the dividend or leave the call option in place you need to find out if:

1. The return from investing the dividend is greater than
2. The financing cost of paying the exercise price X

E.G. On September 5^{th} Wal-Mart Stock is selling for \$47/Share. The Company announces a \$2.50 dividend on December 15^{th} to Shareholders of record on November 15^{th} (ex dividend date). You own a February 45 Call which expire on the 15^{th} of the month. T-Bill Rates are as follows

	Maturity Date			
T-Bills	Nov. 15^{th}	Dec. 15^{th}	Feb 15^{th}	} positive yield Curve }
	4.45%	4.55%	4.65%	

Is it to your advantage to exercise the Wal-Mart call early?



① Cost of financing the \$45 Strike price

Calculate r' $\quad (1+.0465)^{163/365} = (1+.0455)^{101/365}\,(1+r')^{62/365}$

$$\frac{1.020505}{1.012388} = (1+r')^{62/365}$$

$$\left[1.008018\right]^{365/62} = (1+r')$$

1.048137 $\quad = (1.008018)^{5.887097} = 1+r'$

$\Rightarrow r' = 4.81\%$

Calculate r"

\# day between 9/05/2005 and 11/14/2005
Date arithmetic $\Longrightarrow$ 70 days.

$$(1.0465)^{163/365} = (1.0445)^{70/365}(1+r'')^{93/365}$$

$$1.020505 = 1.008385\,(1+r'')^{93/365}$$

$$1.0480 = (1.012019)^{3.924731} = 1+r''$$

$$r'' = \underline{\underline{4.80\%}}$$

Cost of financing the \$45 Strike price to gain the stock at expiration

$$\$45\left[e^{(.0480)(93/365)} - 1\right] = \$45\left(e^{.012817} - 1\right)$$

$$= \$45\,(1.0129 - 1) = 45\,(.012305) = \$.55$$

(2) Return from converting option into stock, securing the dividend and investing it to the expiration date.

$$\$2.50\left(e^{.0481(62/365)}\right) = \$2.50\,e^{.00817} = 2.50\,(1.008204)$$

$$= 2.52 > \$.55$$

Return from Dividend > Cost of financing the strike price

$\Rightarrow$ Exercise the call option early only

If. $(S_t - x) + De^{r't'} > C_t$

Suppose that on November 14th, Walmart Stock is Selling for \$48/Sh. and the Call option is at \$6.50

If you exercise the value of the stock gain plus dividend

$$= (48-45) + 2.52 = 5.52 < 6.50 = C_t$$

Therefore the best course of action would be to stay in the option and not exercise.

Theorem 9 [Since the Call option price decreases as the Strike price increases, intermediate Calls must have value less than or equal to the extreme options]

Given 3 calls on the Same Stock with different exercise prices, the value of the middle option must be less than or equal to the average value of the extreme options.

Maturity	Strike price	Call price
t_1	x_1	C_1
t_2	x_2	C_2
t_3	x_3	C_3

then

$$C_2 \leq \frac{C_1 + C_3}{2}$$

Where $x_1 < x_2 < x_3$
$t_1 < t_2 < t_3$

Texas Instruments Inc. (TXN)

January 12, 2006

At 4:01PM ET: **33.37**

Options

Get **Options** for: GO

View By Expiration: Jan 06 | **Feb 06** | Apr 06 | Jul 06 | Jan 07 | Jan 08

CALL OPTIONS

Expire at close Fri, Feb 17, 2006

	Strike	Symbol	Last	Chg	Bid	Ask	Vol	Open Int
$C_1 \rightarrow$	22.50	TXNBT.X	**11.70**	**0.00**	10.90	11.10	38	38
	25.00	TXNBE.X	**8.90**	**0.00**	8.40	8.60	5	37
	27.50	TXNBY.X	**6.50**	**0.00**	6.00	6.20	7	470
$C_2 \rightarrow$	30.00	TXNBF.X	**3.80**	⇓ .50	3.80	3.90	129	1,263
	32.50	TXNBZ.X	**1.95**	⇓ .36	1.95	2.05	225	3,923
	35.00	TXNBG.X	**0.80**	⇓ .20	0.75	0.85	6,149	13,259
	37.50	TXNBU.X	**0.35**	**0.00**	0.25	0.30	97	1,077
$C_3 \rightarrow$	40.00	TXNBH.X	**0.15**	↑0.05	0.05	0.10	100	36

$$\frac{C_1 + C_3}{2} = \frac{\$11.70 + .15}{2} = \$5.925 > \$3.80 = C_2$$

Arbitrage Theorems on Put Options

The put option conveys the right of the owner to sell the stock at the strike price. It obligates the writer of the put to buy the stock at the exercise (strike) price.

In many ways the put option theorems are mirror images of the call theorems because of the inverse rel= ship that exists between puts and calls.

Theorem 10 : inverse of theorem 1

[A put option will expire with no value (out of the money) or a value equal to amount the strike price exceeds the lower underlying stock price]

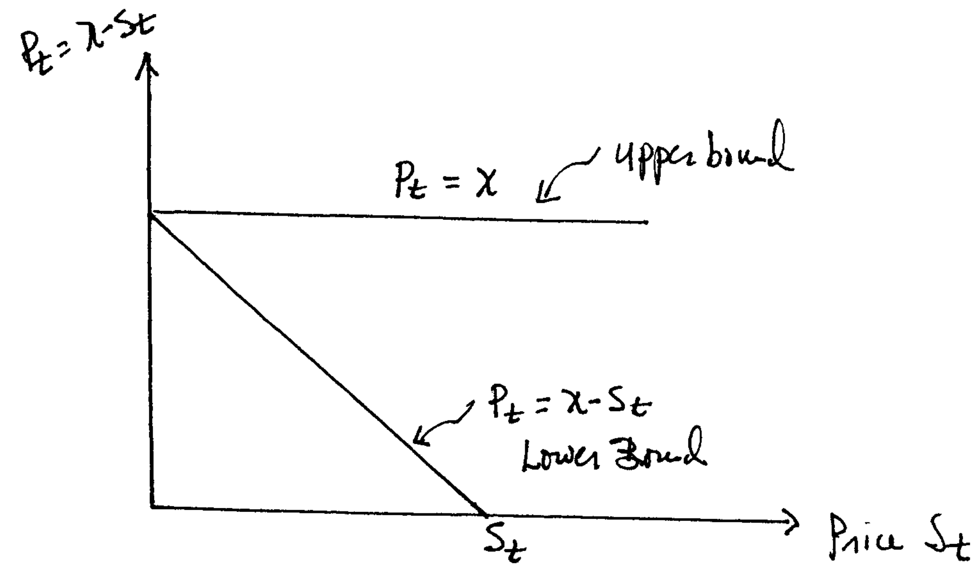
Upon expiration the put premium can be no less than the maximum of either zero or the difference between the exercise price and the stock price.

$$P_t = Max\{0, X - S_t\}$$

- **Theorem II:** [The maximum value of the put is equal to the strike price because the stock price can go no lower than zero.]

The put premium can never be more than the fixed strike price (X).

- $P_t = X - S_t$ let $S_t = 0 \Rightarrow P_t = X$.



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Theorem 12 [Time has value for puts as well as Calls]

Given two options that have the same strike price and the same underlying asset, the option with the longer time to expiration will have a higher premium

$$\text{If } t_1 > t_2, \text{ then } P_1 > P_2.$$

Theorem 13 [putting a stock at a higher price is more valuable to the owner of a put option]

Given two put options that are alike in all respects, except one has a higher exercise price, then it must have a higher put premium.

$$P_t = X - S_t \quad \text{fix } S_t \text{ then } \uparrow P_t \text{ as } X \uparrow$$

- **Theorem 14** [The put premium has an inverse rel&ship with time and risk free yields]

The value of the put at any point in time equals the present value of the strike price [xe^{-rt}] less the underlying value of the stock (when it is in the money) otherwise its 0.

$$P_t \geq Max(0, xe^{-rt} - S_t)$$

- Prior to expiration the put premium will be greater than or equal to either zero or the present value of the exercise price minus the current stock price.

E.G. On January 15, 2006 the price of TXN stock is \$33.03. The following put option is listed on the Yahoo website for the Feb. 2006 put set to expire Feb. 17th 2006.

Strike	Symbol	Ask
40	TXNNH.X	$7.10 = P_t$

- Is the put fairly priced? Given the 30 day T-Bill rate is 4%.

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Sunday, January 15, 2006, 11:41PM ET - U.S. Markets Closed. Dow Nasdaq +0.02%

To track stocks &

GO

Texas Instruments Inc. (TXN)

On Jan 13: 33.03

Options

Get **Options** for: GO

View By Expiration: Jan 06 | **Feb 06** | Apr 06 | Jul 06 | Jan 07 | Jan 08

CALL OPTIONS Expire at close Fri, Feb 17, 2006

Strike	Symbol	Last	Chg	Bid	Ask	Vol	Open Int
22.50	TXNBT.X	**11.70**	**0.00**	10.50	10.70	38	38
25.00	TXNBE.X	**8.22**		8.00	8.20	10	37
27.50	TXNBY.X	**5.60**		5.60	5.80	327	470
30.00	TXNBF.X	**3.40**		3.40	3.50	149	1,276
32.50	TXNBZ.X	**1.75**		1.65	1.75	917	3,934
35.00	TXNBG.X	**0.65**		0.65	0.70	1,093	18,771
37.50	TXNBU.X	**0.20**		0.15	0.25	25	1,083
40.00	TXNBH.X	**0.15**	**0.00**	N/A	0.10	100	130

PUT OPTIONS Expire at close Fri, Feb 17, 2006

Strike	Symbol	Last	Chg	Bid	Ask	Vol	Open Int
22.50	TXNNT.X	**0.05**	**0.00**	N/A	0.05	10	10
25.00	TXNNE.X	**0.05**	**0.00**	N/A	0.05	10	10
27.50	TXNNY.X	**0.10**	**0.00**	0.05	0.15	10	133
30.00	TXNNF.X	**0.40**	↑ 0.20	0.35	0.40	314	1,555
32.50	TXNNZ.X	**1.15**	↑ 0.20	1.05	1.15	527	13,816
35.00	TXNNG.X	**2.65**	↑ 0.45	2.50	2.60	43	1,115
37.50	TXNNU.X	**4.60**	↑ 0.70	4.50	4.70	12	216
40.00	TXNNH.X	**6.00**	**0.00**	6.90	7.10	10	10
42.50	TNZNV.X	**9.50**	↑ 1.00	9.40	9.60	8	8

Highlighted options are in-the-money.

Expand to Straddle View...

2%
rack stocks &

33.03

- Calculate: $40e^{-.04(30/365)} - 33.03$

$= 40e^{-.003288} - 33.03$

$= 40(.996718) - 33.03 = 39.87 - 33.03$

$= \$6.84$

$P_t = \$7.10 > \$6.84$ So, the put is fairly priced.

Theorem 15 [The relevant risk of the put is the risk of the underlying Stock]

- The higher (lower) the risk of the underlying asset, the higher (lower) the put premium.

Theorem 16 [The present value of the exercise price is always less than the exercise price]

- When the Stock does not pay a dividend during the life of a put option then early exercise may be optimal at any time.

Sunday, January 15, 2006, 11:41PM ET - U.S. Markets Closed. Dow Nasdaq +0.02%

To track stocks &

Quotes & Info Enter Symbol(s): GO Symbol Lookup | Finance Search

Texas Instruments Inc. (TXN)

On Jan 13: **33.03**

Options

Get **Options** for: GO

View By Expiration: Jan 06 | **Feb 06** | Apr 06 | Jul 06 | Jan 07 | Jan 08

CALL OPTIONS Expire at close Fri, Feb 17, 2006

Strike	Symbol	Last	Chg	Bid	Ask	Vol	Open Int
22.50	TXNBT.X	**11.70**	**0.00**	10.50	10.70	38	38
25.00	TXNBE.X	**8.22**		8.00	8.20	10	37
27.50	TXNBY.X	**5.60**		5.60	5.80	327	470
30.00	TXNBF.X	**3.40**		3.40	3.50	149	1,276
32.50	TXNBZ.X	**1.75**		1.65	1.75	917	3,934
35.00	TXNBG.X	**0.65**		0.65	0.70	1,093	18,771
37.50	TXNBU.X	**0.20**		0.15	0.25	25	1,083
40.00	TXNBH.X	**0.15**	**0.00**	N/A	0.10	100	130

PUT OPTIONS Expire at close Fri, Feb 17, 2006

Strike	Symbol	Last	Chg	Bid	Ask	Vol	Open Int
22.50	TXNNT.X	**0.05**	**0.00**	N/A	0.05	10	10
25.00	TXNNE.X	**0.05**	**0.00**	N/A	0.05	10	10
27.50	TXNNY.X	**0.10**	**0.00**	0.05	0.15	10	133
30.00	TXNNF.X	**0.40**	↑ 0.20	0.35	0.40	314	1,555
32.50	TXNNZ.X	**1.15**	↑ 0.20	1.05	1.15	527	13,816
35.00	TXNNG.X	**2.65**	↑ 0.45	2.50	2.60	43	1,115
37.50	TXNNU.X	**4.60**	↑ 0.70	4.50	4.70	12	216
40.00	TXNNH.X	**6.00**	**0.00**	6.90	7.10	10	10
42.50	TNZNV.X	**9.50**	↑ 1.00	9.40	9.60	8	8

Highlighted options are in-the-money.

Expand to Straddle View...

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Searc

03

ES. Assume TXN Sells for 33.03, the 40 put expires in 30 days, the one month T-Bill rate is 4% ⇒ the minimum premium is $6.84.

If the put is exercised the return would be : $40 - $33.03 = $6.97

The bid price on the put option is $ 6.90

Therefore, you would be slightly better off exercising the option: (by $6.97 - $6.90 = $.07).

Theorem 17 [when considering the exercise of the put you need to consider the value of the dividend in relation to the return on receiving the Strike price]

When a stock pays a dividend during the life of the put, then early exercise **will not** be optimal if the dividend plus interest exceeds the interest that could be earned on the exercise (Strike) price.

$$D e^{r't'} > X\{e^{r''t''} - 1\} \Rightarrow \text{not optimal to exercise}$$

Theorem 18 [Pricing Consistency between near & far put options]

Given three put options on the same stock with different exercise prices, the value of the middle put must be less than or equal to the average value of the extreme options.

time	Strike price	Put Price
1	x_1	P_1
2	x_2	P_2
3	x_3	P_3

$$P_2 \leq \frac{P_1 + P_3}{2}$$

- Arbitrage Rel=ships Between Put & Call Options

Theorem 19 {Put-Call Parity}

Given a European put and call option, the value of the put must equal the value of a like call plus the present value of the exercise price minus the stock price.

$$P_t = C_t + Xe^{-rt} - S_t$$

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Explanation:

$$\underbrace{P_t - C_t}_{\text{Difference in value between selling versus buying the stock at strike price } X} = \underbrace{Xe^{-rt} - S_t}_{\text{Difference between the present value of the strike price less the stock price}}$$

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Using the Put-Call Parity Equation to determine Appropriate Put price:

Given the following information on Intel:

Nov 75 Intel Call premium = \$6.875

Stock price on Intel = \$ 77.50

60 day T-Bill rate = 4.8%

Today's Date: Sept. 28th 2005

Time period: $\underbrace{9/28/2005 \longrightarrow 11/15/2005}_{48\ \text{days}}$

$$P_t = C_t + Xe^{-rt} - S_t$$

$$P_t = 6.875 + 75e^{-.048(48/365)} - 77.50 = \$3.90$$

In order for no arbitrage to occur the put must sell for \$3.90.

Application

- What if the the put is over valued. at \$5.00?

Call premium Nov 75 on Intel = \$6.875
Put premium Nov 75 on Intel = \$5.00
Stock Price = \$77.50

If the put is over valued – you need to sell (Short) the put, Short the Stock, and buy the Call.

Trade

Sell the put	\$5.00
Short the Stock	\$77.50
Buy the Call	(\$6.875)
Net	\$75.625

Expiration Day

\$79
Stock Price > \$75

Net.	75.625
Use Call to cover the Short	(75)
Ending Net position	.625

\$73
Stock Price < \$75

Net.	75.625
Call out of the money Buy Stock to cover Short	(73)
Loss on put (73-75) =	(2)
Ending Net position	.625

Theorem 20 Given an American put and call option, the value of the put will be greater than or equal to the value of the call plus the present value of the strike price less the stock price

$$P_t \geqslant C_t + Xe^{-rt} - S_t$$

$\Rightarrow$ for American put options there can be no definitive pricing model. (i.e. inequality vs. equality for the European put)

Theorem 21 If the underlying stock pays dividends during the life of the put option, then the dividend will increase the value of the put option, by the amount of dividend accumulated with interest.

$$P_t \geqslant C_t + D_t e^{rt} + X e^{-rt} - S_t$$